

DEVELOPING YOUNG CHILDREN'S UNDERSTANDING OF PLACE-VALUE USING MULTIPLICATION AND QUOTITIVE DIVISION

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This paper focuses on selected findings from a study that explored the use of multiplication and division with 34 five- and six-year-old children from diverse cultural and linguistic backgrounds. The focus of instructional tasks was on working with groups of ten to support the understanding of place value. Findings from relevant assessment tasks and children's work highlighted the importance of encouraging young children to move from unitary (counting by ones) to tens-structured thinking.

BACKGROUND

This study has emerged from the findings from both the national numeracy results (Young-Loveridge, 2010) and recent international results (May, 2013). The latest results of the Programme for International Student Assessment (PISA) (a study that assesses and compares how well countries are preparing their 15-year-olds to meet real-life opportunities and challenges) showed that New Zealand's average scores in mathematics have declined since 2009. Compared to countries with a similar average score, New Zealand has a larger proportion of students who can complete only relatively basic mathematical tasks (below Level 2), as well as students who are capable of advanced mathematical thinking and reasoning (Level 5 and above).

Mathematics reform over the past few decades has led to the development of frameworks outlining progressions in number as students acquire increasingly sophisticated ways of thinking and reasoning (Bobis, Clarke, Clarke, Thomas, Wright, Young-Loveridge, & Gould, 2005). Typically, at the lower stages, students solve problems by using counting strategies. As they come to appreciate additive composition, they are able to use strategies that involve partitioning and recombining quantities (part-whole thinking). The initial focus with younger children is often on addition and subtraction before introducing other domains such as multiplication and division, and proportional reasoning. Students are thought to need particular number knowledge in order to apply strategies for solving problems (Ministry of Education, 2008). Such knowledge includes number-word sequences, basic facts, and place value.

Our numeration or place-value system is characterised by four key properties: positional, base-ten, multiplicative, and additive (Ross, 1989). Place value is considered to be an essential foundational concept in mathematics. Fuson, Smith, and Cicero (1997) present a model of two-digit conceptions arranged developmentally, from unitary (count by ones) through an understanding of the ten-based structure, to a multi-unit conception. The shift from unitary counting to these higher stages involves

developing an understanding of part-whole relationships. Thompson (2000) argues that place value is too sophisticated for many young children to grasp and this idea is supported by evidence showing that young children have difficulty understanding the place-value system (Kamii, 1988; Ross, 1989). However, others have shown that with carefully planned learning experiences, first grade students can learn the beginnings of place value structure (e.g., Kari & Anderson, 2003; van de Walle, Karp, Lovin, & Bay-Williams, 2014). Mulligan and Mitchelmore's (2009) innovative work on promoting awareness of pattern and structure is consistent with this approach. Grouping and partitioning activities can lay the foundations for developing place value, beginning with tens and ones and extending beyond two digits. Partitioning small numbers, composing wholes from parts, rearranging parts while recognising that the quantity of the whole has not changed, all contribute to developing an understanding of place value and part-whole relationships (Ross, 1989).

It is important for students to develop both counting-based and collections-based approaches to working with numbers (Yackel, 2001). Yang and Cobb (1995, p. 10) have highlighted "an inherent contradiction" in the way that Western children are initially encouraged to count by ones and thus construct unitary *counting-based* number concepts, but are then expected to reorganise these into *collections-based* concepts involving units of ten and units of one when place-value instruction begins. Yang and Cobb contrast the Western counting-based view with the collections-based approach of Chinese mothers and teachers, who emphasize groups (units) of ten. The difference in emphasis on counting versus grouping by tens helps to explain Yang and Cobb's (1995) finding of more advanced mathematical understanding by the Chinese children relative to that of the American children.

The challenge of learning about place value is evident when students in the middle grades show limited understandings of two-digit numbers (Ross, 1989). Language factors have also been shown to influence place value understandings in different cultures. Asian language speakers, for example, have been shown to have a better understanding of place value than English language speakers. The irregularities and inconsistencies in the English language (e.g., '-teen' & '-ty' numbers) contrast with the transparent patterns found in most Asian languages, and research shows more advanced development of place-value understanding in Asian children (Miura, Okamoto, Kim, Steere, & Fayol, 1993).

Although many western mathematics curricula introduce place value before multiplication and division, it has been suggested that multiplication and division provide an important conceptual foundation for understanding place value (Carpenter, Fennema, Franke, Levi, & Empson, 1999; Ross, 1989; van de Walle et al., 2014). Carpenter et al., emphasize the particular importance of quotitive (measurement or repeated subtraction) division problems that require objects to be collected into groups of ten to help develop the base-ten concept.

Place value is a key aspect of number sense, defined as the "understanding of number and operations along with the ability and inclination to use this understanding in

flexible ways” (McIntosh, Reys & Reys, 1992, p. 3). A framework for the development of number sense outlined by McIntosh et al., includes knowledge and facility with numbers. Within this component there are four key aspects: a sense of orderliness of numbers (patterns and regularities); multiple representations for numbers (symbols &/or graphical representations); a sense of relative and absolute magnitude of numbers; and a system of benchmarks. Place value is a component of the sense of orderliness of numbers. This framework positions place-value understanding within a broader context and highlights its importance for children learning to engage in mathematical thinking.

The project described here set out to explore the impact of using multiplication and division contexts with five- and six-year-olds on their emerging understandings of number, including part-whole relationships and place value.

THE STUDY

This study was set in an urban school (medium SES) in New Zealand. The participants were 34 five- and six-year-olds (17 girls & 17 boys) in two classes, one designated as Year 1 and the other Year 2. The average age of the students was 6.2 years at the beginning of the study (range 5.6 to 6.9 years). The children were from a diverse range of ethnic backgrounds, with approximately one third of European ancestry, one third Māori (the indigenous people of New Zealand), and other ethnicities including Asian, African, and Pasifika (Pacific Islands people). One third of the children had been identified as English Language Learners [ELL]. At the start of the study, the children were assessed individually using a diagnostic task-based interview designed to explore their number knowledge and problem-solving strategies (April). The assessment interview was completed again after each of the two four-week teaching blocks (June and November). The assessment tasks included: addition, subtraction, multiplication, division, basic facts, incrementing in tens, counting sequences, and place value.

Teaching using Multiplication and Division Contexts

Two series of 12 focused lessons were taught; the first phase was in May and the second in October. In these lessons the children were introduced to groups of two, using familiar contexts such as pairs of socks, shoes, gumboots, jandals, and mittens. Multiplication was introduced using simple word problems, such as:

Kiri, Sam, and Len each get 2 socks from the bag. How many socks do the 3 children have altogether?

Once children were familiar with working with groups of two, groups of five were introduced using contexts such as gloves focusing on the number of fingers on each glove, and five candles on a cake. The next objective was to introduce groups of ten. For this the context of filling cartons with eggs was introduced with cartons that held exactly ten eggs. Although the emphasis of the study was on multiplication and division, the focus in this paper is specifically on the quotitive division problems, making groups of ten, and considering leftover ones.

A typical problem was:

There are 23 eggs. Each carton holds 10 eggs. How many full cartons are there?

Later problems were posed so that children could self-select numbers, including generating their own ‘mystery number’ inside the empty brackets.

There are 27 [76] [] chocs. Each box holds 10 chocs. How many full boxes are there?

Lesson Structure

A typical lesson began with all students completing a problem together on the mat, using materials to support the modelling process, and sharing ways of finding a solution. The teacher recorded children’s problem-solving processes (including use of manipulatives) and discussion in a large scrapbook (‘modelling book’). The problem for the day was already written in the book and both drawings and number sentences were recorded, acknowledging individual children’s contributions. The children then completed a problem in their own project books, choosing a similar or larger number, and/or selecting a new number. Materials (egg cartons and unifix cubes) were made available and children were encouraged to show their thinking using representations and to record matching equations.

RESULTS

Children’s performance on the tasks was examined to look for patterns and progressions. A tens-structure sub-score was calculated using students’ responses on 24 tasks related to working with groups of ten (e.g., 60 sticks for grouping into tens; known facts such as $20+7$, $10+8$, $10+10$, 2×10 , $60\div 10$, $80\div 10$, $200\div 10$, $23\div 10$, half of 20; \$10 notes in \$80, \$240; the meaning of “2” in “25”; incrementing in tens such as 15 and 10, 42 and 30; and producing quantities using groups of ten). Children’s responses to addition, subtraction, and multiplication problems were weighted according to the sophistication of strategies (counting all = 1, counting on/back or in multiples = 2, known & derived facts = 3).

Children’s tens-structure sub-scores ranged from 0 to 24 (1 per task). A comparison of the top 20% of the distribution ($n=6$) with the bottom 20% ($n=6$) showed a marked difference in their knowledge of number and relationships, key ideas for tens-structured thinking. The lowest performers were able to complete no more than one task, whereas the top performers completed between 15 and 24 tasks successfully. These children with stronger baseline knowledge of facts, number –word sequences, and counting strategies (e.g., counting on/back), progressed to skip counting and using known or derived facts. They were fluent with incrementing by tens (adding), and also working with multiples of ten (multiplying & dividing). These six children were able to recognise that six groups of ten could be made from 60 objects, justifying their responses by referring to the tens digit.

Children’s performance improved on many of the tasks related to tens-structured thinking. For example, when shown an array of 30 cakes in three rows of ten and asked

how many cakes altogether, the majority of children (85%) could work out the answer by the end of the project, an increase from 32%. Approximately one-quarter (24%) of these students used known or derived facts, and more than half (56%) used skip counting. More than half of the children were able to combine a multiple of ten (a ‘-teen’ or a ‘-ty’ number) with a single-digit quantity without using a counting strategy. For example, 62 per cent knew $20 + 7 = 27$ and 53% knew that $10 + 8 = 18$. Children were shown a bag of 60 sticks (labelled with its total) and a bundle of ten sticks, and asked how many bundles could be made from the bag of sticks. Not quite half (44%) of the children were able to work out the answer by looking at the number ‘60’. More than half of the children (59%) knew the number of \$10 notes needed to buy an \$80 toy, up from 12 per cent initially. Almost one third (32%) were able to work out the \$10 notes needed for an item costing \$240. When similar tasks were presented using symbolic expressions as known facts, fewer were able to respond correctly (e.g., 32% knew $60 \div 10$ and $80 \div 10$, while 24% knew $200 \div 10$). One of the most difficult tasks was showing the meaning of the ‘2’ in ‘25’ for a picture of 25 blocks where the two groups of ten were linked. Only nine children (29%) circled the two groups of ten blocks rather than two single blocks. Table 1 presents the inter-correlations for responses to selected tasks at the start and end of the project.

	Nov Add/Sub	Nov Mult	Nov Div	Nov Facts	Nov Tens	Apr A/S	Apr Mult	Apr Div
Nov Mult	0.46							
Nov Div	0.71	0.36						
Nov Facts	0.75	0.66	0.72					
Nov Tens	0.74	0.54	0.75	0.96				
Apr A/S	0.80	0.63	0.68	0.75	0.72			
Apr Mult	0.70	0.50	0.55	0.67	0.59	0.73		
Apr Div	0.65	0.52	0.65	0.71	0.69	0.62	0.61	
Apr Facts	0.83	0.63	0.73	0.86	0.83	0.81	0.73	0.69

Table 1: Correlations among tasks including tens-structured tasks (Nov Tens)

At the start of the study, knowledge of known facts was strongly related to the score on addition and subtraction problem solving ($r = 0.81$). A similar relationship was found for solutions to multiplication ($r = 0.73$), and division problems ($r = 0.69$). Correlations from the start to the end of the project (inside the bordered region) indicate that knowledge of known facts were most predictive of subsequent strategies for addition and subtraction (0.83), and division (0.73). They also predicted the measure of tens-structured awareness (0.83).

Children’s representations from both an assessment task and the project books provide evidence that they can represent 2-digit numbers as groups of ten and ones. In the assessment interview, two-thirds (68%) of the children drew accurate diagrams representing 23 eggs in cartons of ten. In the children’s individual project books, they constructed their own ways of showing their thinking. However, they were encouraged

to use a ten-frame as a representation for quotitive division problems using egg cartons. Figure 1 shows how Nisha solved the following problem:

There are 59 eggs. Each carton holds 10 eggs. How many full cartons are there?

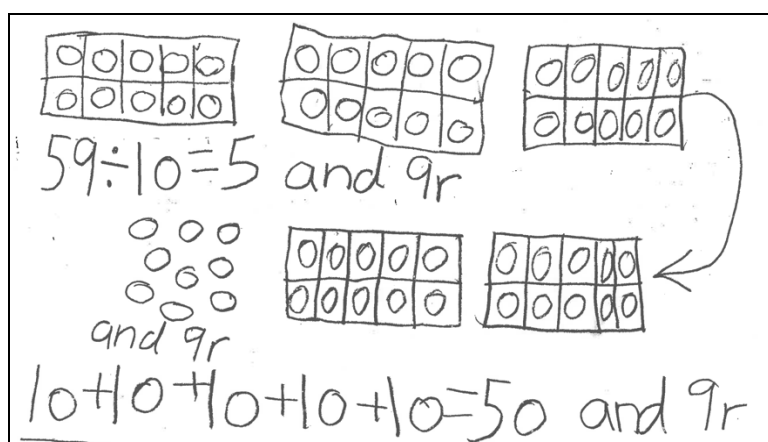


Figure 1: Student's representation of division word problem

Nisha's work shows her clear understanding of groups of ten displayed in ten-frames and recognition of nine ones units as a remainder (9r). Some children referred to these as 'leftovers' whereas others readily adopted the convention of recording this as 'r'. Her second equation was in response to children being asked to justify their solution. In this instance, Nisha has chosen to use an addition equation to show her thinking rather than multiplication.

DISCUSSION

The findings of this study show that even children as young as five and six years of age are able to work with multiplication and division problems, as well as place-value tasks. This is consistent with the work of researchers advocating for the introduction of place-value in the early years (e.g., Fuson et al., 1997; Kari & Anderson, 2003; van de Walle et al., 2014). This provides evidence contrasting with Thompson's (2000) caution about introducing place value to young children, and also challenges international curricula that introduce place value before multiplication and division.

Children were provided with opportunities to solve problems using different contexts (e.g., egg cartons and chocolate boxes) and manipulatives (e.g., unifix cubes). They were also encouraged to use representations such as ten-frames to show their thinking. This enabled children to represent groups of ten as composite units. They were also supported in recording their solutions as equations. The language of place value (e.g., '-teen' and '-ty' numbers) was challenging for the children, consistent with research findings (e.g., (Miura et al., 1993; Yang & Cobb, 1995). One-third of the sample was composed of English language learners. However, it is difficult to know whether these children were advantaged or disadvantaged in learning about place value. Interestingly, three of the top six performers were ELLs whose first languages have transparent tens-structure.

The introduction of multiplication and division prior to formal place-value instruction was beneficial to the students, not just in understanding multiplication and division, but also in developing place-value understanding. The children's knowledge of tens-structure reflected in their recall of known facts and working with groups of ten, is consistent with Yang and Cobb's (1995) argument about the need to move from a counting-based to a collections-based approach for place-value understanding. This assists the transition from counting strategies to part-whole thinking. Providing opportunities to work with 2-digit numbers meant that several children self selected larger numbers (3-digit) for their word problems, moving well beyond expectations at this level (Ministry of Education, 2009).

The early recognition of the underlying patterns and structure of groups of ten in 2-digit numbers provided a foundation from which some children were able to abstract and generalise to larger numbers (Mulligan, 2010; Mulligan & Mitchelmore, 2009; McIntosh et al., 1992). The fact that only a few children were able to demonstrate the meaning of the '2' in '25' indicates the challenge of building a sound understanding of place value. However, this exploratory study has shown that learning experiences using multiplication and quotitive division problems contributes to the development of place-value understanding.

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